Numerical Investigation of Turbulence Models for Shock-Separated Boundary-Layer Flows

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Abstract

ALGEBRAIC and kinetic energy models of turbulence are used to describe the Reynolds shear stress in numerical solutions of the Navier-Stokes equations for shock-separated turbulent boundary-layer flows. The models are assessed by the ability of the numerical solutions to predict physical phenomena of two extensively documented experiments. Calculated values of skin friction, wall pressure distribution, and heat transfer are compared with measurements.

Contents

To simulate two experimental flows involving shock-separated turbulent boundary layers, both algebraic eddy viscosity (zero-equation) and kinetic energy of turbulence (one-equation) models are investigated. These models are incorporated into the time-averaged compressible Navier-Stokes equations, which are then solved numerically.

The experiments were conducted at the NASA Ames Research Center. The first of these (Fig. 1) consists of a transonic normal shock-wave boundary-layer interaction (TSBLI) in a circular duct at an upstream Mach number of about 1.5.[1] The second experimental flow (Fig. 2) consists of an oblique shock-wave boundary-layer interaction in an axially symmetric hypersonic flow (HSBLI) at a Mach number of about 7.[2] The documentation for these flows includes surface measurements and mean and fluctuating flowfield measurements.

The differential equations used to describe the mean flow for this study are the time-dependent, mass-averaged Navier-Stokes equations for axially symmetric flow of a compressible fluid. The turbulent Reynolds stress and heat-flux terms in these equations are assumed to be related to the mean-flow gradients of velocity and temperature by means of eddy transport coefficients that are simply added to the molecular transport coefficients. Additional restrictions on the differential equations include the perfect gas assumption, constant specific heats, the Sutherland viscosity law, and zero bulk viscosity. The resulting equations for the conserved quantities ρ, ρu, ρv, ρe, and ρk, where ρ is the time-averaged density, u and v the mass-averaged velocity components, e the total specific energy per unit mass, and k the specific kinetic energy of turbulence, are given explicitly in Ref. 3.

The numerical procedure used is the basic second-order, predictor-corrector, finite-difference, time-splitting method of MacCormack[2] modified by the new computer algorithm in Ref. 4. This is the first application of this new algorithm to a flow in which the turbulence is described by a one-equation model. The modifications apply near the surface. There the equations are time-split into a hyperbolic part and a parabolic part. The hyperbolic part is solved using a new explicit numerical method based on characteristics theory; the parabolic part is solved by an implicit method. In comparison to the unmodified method, this new numerical method dramatically reduces the computation time required to solve the time-dependent Navier-Stokes equations for compressible flows at high Reynolds numbers. Details of the computational grids and boundary conditions used are described in Refs. 1-3.

The expressions for the total viscosity coefficient, total thermal conductivity, total kinetic energy diffusivity, and the pressure are, respectively,

$$\mu_T = \mu + \mu_t, \quad \mu_k = \mu + \mu_{tk}, \quad K_T = K + K_t, \quad P_T = p + 2/3\rho k \quad (1)$$

where μ is the molecular viscosity; K is the thermal conductivity, K is related to the molecular Prandtl number by $Pr = \mu c_p / K$; and p is the hydrostatic pressure. The remaining terms on the right-hand side of Eq. (1) depend on the turbulence model and are defined later.

For the zero-equation models, the equations solved do not include the turbulence variable ρk (thus μ_k is not included)

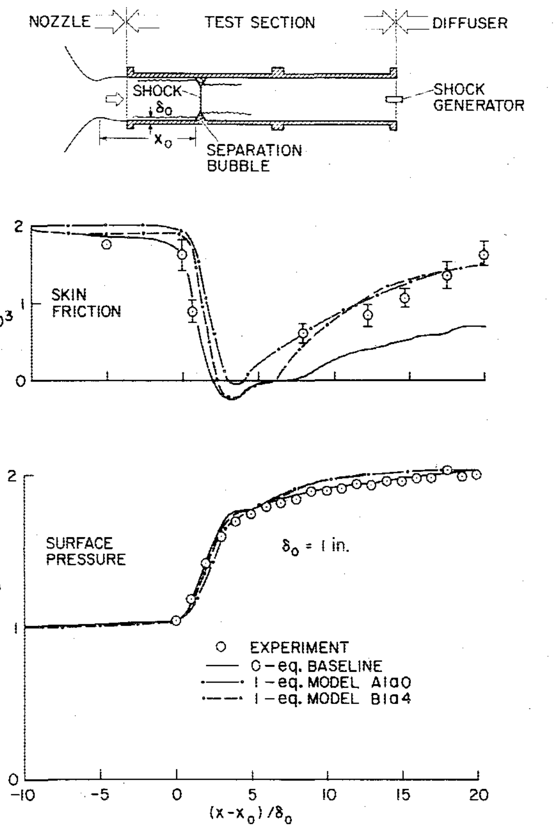

Fig. 1 Comparison of measured and computed skin-friction and wall-pressure distributions for the normal shock/boundary-layer interaction experiment ($Re_{x_0} = 3.67 \times 10^7$, $M_\infty = 1.44$, $p_\infty = 5.94$ psia, $T_\infty = 360.5$ °R).

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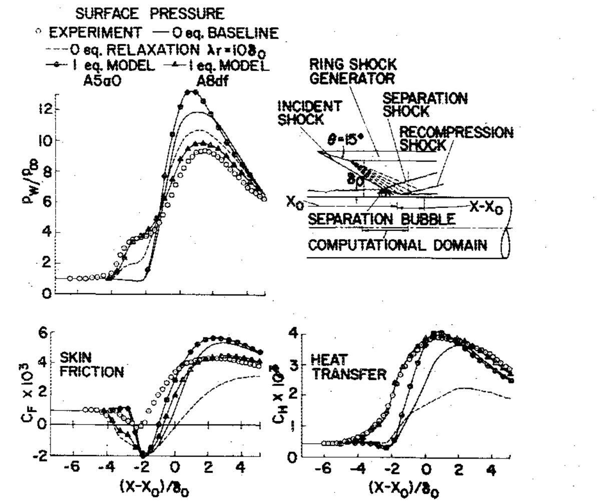

Fig. 2 Comparison of measured and computed skin-friction and heat-transfer distributions for hypersonic shock/boundary-layer interaction ($Re_{x0} = 1.3 \times 10^7$, $M_\infty = 6.85$, $\alpha = 15$ deg, $p_\infty = 0.089$ psia, $T_\infty = 120\,°R$).

and the eddy diffusivities are modeled algebraically. The eddy diffusivity for thermal conduction is related to μ_t and the turbulent Prandtl number by $Pr_t = \mu_t c_p / K_t$; for these calculations, Pr and Pr_t are taken as 0.72 and 0.9, respectively. The eddy viscosity μ_t is expressed in terms of an inner and outer eddy viscosity function

$$\mu_t = \begin{cases} \mu_{t_{\text{inner}}} = \rho \ell^2 \, |\partial u/\partial y + \partial v/\partial x| & y \le y_c \\ \mu_{t_{\text{outer}}} = 0.0168 \rho u_e \delta^* / [1 + 5.5 (y/\delta)^6] & y > y_c \end{cases} \tag{2}$$

where y_c is the first point at which $\mu_{t_{\text{inner}}}$ exceeds $\mu_{t_{\text{outer}}}$. The function ℓ is the Prandtl mixing length modified by the van Driest damping factor.[3] δ^* is the displacement thickness for an incompressible fluid and δ is the boundary-layer thickness.

For an upstream relaxation zero-equation model, the local equilibrium (baseline) eddy viscosity functions, Eq. (2), are modified to include effects of previous history on the flow through a relaxation length $\lambda_r = c_r \delta_0$, where c_r is an arbitrary constant and δ_0 is the upstream boundary-layer thickness.

The one-equation model used here is the one developed and applied to incompressible flow by Glushko[5] and recently extended to compressible flows by Rubesin.[6] In this model, the eddy diffusivities are functions of the Reynolds number of the turbulence, $r_t \equiv \sqrt{k}\,\rho L/\mu$, and are given explicitly by

$$\mu_t = \mu \epsilon(r_t)$$

$$K_t = K Pr \epsilon(\Gamma r_t) \quad \text{or} \quad K Pr \Gamma \epsilon(r_t)$$

$$\mu_{tk} = \mu \epsilon(\lambda r_t) \quad \text{or} \quad \mu \lambda \epsilon(r_t) \tag{3}$$

where

$$\epsilon(r_t) = \alpha r_t \begin{Bmatrix} r_t/r_0 \\ r_t/r_0 - (r_t/r_0 - 0.75)^2 \\ 1 \end{Bmatrix}$$

$$\text{for} \begin{Bmatrix} 0 \ge r_t/r < 0.75 \\ 0.75 \le r_t/r_0 < 1.25 \\ 1.25 \le r_t/r_0 < \infty \end{Bmatrix} \tag{4}$$

and where α, Γ, λ, and r_0 are constants. The length scale of the turbulence L, as defined by Glushko, is

$$L = \delta \begin{Bmatrix} y/\delta \\ (y/\delta + 0.37)/2.61 \\ (1.48 - y/\delta)/2.52 \end{Bmatrix} \text{for} \begin{matrix} 0 \le y/\delta < 0.23 \\ 0.23 \le y/\delta < 0.57 \\ 0.57 \le y/\delta \le 1.48 \end{matrix} \tag{5}$$

In Table 1 of Ref. 3 some of the values of the constants that were tried in this study are listed, as well as the form chosen for the eddy diffusivities, Eq. (3).

Representative results for the TSBLI and HSBLI problems are shown in Figs. 1 and 2, respectively. In these figures, results of zero-equation turbulence models and particular one-equation models are compared with experimental measurements. The various one-equation turbulence models used in these figures are identified by an alphanumeric code which is defined in Ref. 3. For purposes of this synoptic, model A1ao is the original Glushko-Rubesin model, model A5ao uses the alternative form of the eddy diffusivities given in Eq. (3), and models B1a4 and A8df correspond, respectively, to mild and extensive length-scale modifications which lead to better agreement with experimental results. Other variants utilized by the authors are reported in Ref. 3.

Comparisons of calculated and experimentally determined skin friction and wall pressure distribution for the TSBLI experiment are shown in Fig. 1. It is apparent from this figure that both zero- and one-equation model solutions agree well with the measured pressure distribution. However, the measured rise in skin friction downstream of reattachment is captured significantly better by the original Glushko-Rubesin one-equation model (A1ao) than by the zero-equation model. The length-scale parameters of the one-equation model were found to have an important effect on the size of the separated region for the TSBLI problem (B1a4). In addition, the parameters α, r_0, λ, and the coefficient of the dissipation term in the equation for the kinetic energy of turbulence had a strong influence on the magnitude of the skin friction upstream of separation.

Comparisons of calculated and experimentally determined wall pressure, skin friction, and heat transfer for the HSBLI experiment are shown in Fig. 2. This flow had large compressibility effects associated with it and provided a severe test of the models. Initial comparisons indicated that all turbulence models had difficulty predicting this flow successfully (see zero-equation models and one-equation model A5ao results on this figure). Subsequent extreme length-scale modifications for the one-equation model were tried to obtain better predictions of pressure distribution and heat transfer and culminated in one-equation model A8df. This model agreed fairly well with skin friction data as well, especially downstream of reattachment. Perhaps even better skin friction predictions could be obtained with further length-scale modifications, but this is a tedious process and is not presently being pursued. Instead, more advanced turbulence models which generate their own length scales (i.e., two-equation models) are being studied and will be the subject of subsequent papers.

References

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